ROTATION OF THE SOLAR INTERIOR: NEW RESULTS BY HELIOSEISMIC DATA INVERSIONS

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ABSTRACT

We determine the variation of the Sun's angular velocity with latitude and radius by means of a helioseismic inversion of more than 30,000 of p-mode splitting coefficients. These data were obtained from the first set of uninterrupted Doppler images from SOI-MDI (on board the SOHO satellite) in 1996 (Schou et al. 1994), which yield splittings of great accuracy, never obtained in previous sets of data. This allow us to probe rotation in the convection zone and the outer part of the radiative core, and obtain more reliable results closer to the poles (Schou et al. 1998). We are also able to study rotational details in the shear layer (*tachocline*) located near the base of the convection zone and determine the behaviour of the angular velocity in the solar core. Unfortunately, the data uncertainties for low ℓ degrees result of weak significance in a rotational profile in the deepest layers. In order to infer the rotation of the deep interior accurately, we also invert some sets of data with more accurate splittings of the lowest degree modes ($\ell \sim 1-4$). Currently, the best data sets for this purpose still come from groundbased networks such as, BiSON, IRIS and GONG (Chaplin et al. 1996; Lazreck et al. 1996, Gavryuseva & Gavryusev 1998). Finally, since these inversions, based on different low-ℓ data still give conflicting solutions in the core, we discuss alsothe results obtained by inverting the latest accurate low-ℓ data of the GOLF instrument on SOHO.

Key words: helioseismology; solar dynamics.

1. ROTATIONAL SPLITTINGS

The rotation breaks the spherical symmetry of the solar structure and splits the frequency of each oscillation mode of harmonic degree ℓ into $2\ell+1$ components. Since rotation acts as a small perturbation, it can be shown that the rotational splitting for each mode of harmonic degree ℓ, radial order n and azimuthal order m, is related to the rotation rate $\Omega(r,\theta)$ inside the Sun by:

$$\Delta\omega_{n,\ell,m} = \int_0^{R_\odot}\int_0^{\pi} \mathcal{K}_{n,\ell,m}(r,\theta)\Omega(r,\theta) r dr d\theta \quad (1)$$

where r is the radial distance from the centre and θ is the co-latitude. $\mathcal{K}_{n,\ell,m}(r,\theta)$ are the mode kernel functions which depend on the equilibrium model quantities and oscillation eigenfunctions. The dependence of splittings on angular velocity can be used in a 2-dimensional inverse problem to probe the Sun's internal differential rotation.

Generally, the observations have not been applied in terms of individual mode frequencies, but rather as so-called a-coefficients (Schou, Christensen-Dalsgaard, Thompson 1993). Using these a-coefficients, the mode frequencies could be expanded in polynomials:

$$\omega(n,\ell,m) = \omega(n,\ell) + 2\pi \sum_{j=0}^{jmax} a_j(n,\ell)\mathcal{P}_j^{(\ell)}(m)$$

where the polynomials $\mathcal{P}_j^{(\ell)}(m)$ have been chosen in the same way as Ritzwoller and Lavely (1991). Because of the symmetry properties of the splittings, the solar rotation is described only by the odd coefficients a_j. It follows that the expansion of the splittings in polynomials corresponds to an expansion of $\Omega(r,\theta)$ such that:

$$\Omega(r,\mu) = \sum_{j=0}^{jmax} \widetilde{\Omega}_{2j+1}(r)\frac{dP_{2j+1}(\mu)}{d\mu} \quad (2)$$

where $P_{2j+1}(\mu)$ are the Legendre polynomials with $\mu = cos\theta$. So, the a-coefficients are related to the expansion functions $\widetilde{\Omega}_{2j+1}(r)$ by:

$$2\pi a_{2j+1}(n,\ell) = \int_0^{R_\odot} \mathcal{K}_{n,\ell,j}(r)\widetilde{\Omega}_{2j+1}(r)dr \quad (3)$$

in which the kernels are calculated according to:

$$\mathcal{K}_{n,\ell,j}(r) = [F_{1,n,\ell} - F_{2,n,\ell}(2j+3)j]v_{2j+1,\ell} \quad (4)$$

where

$$F_{1,n,\ell} = [\xi_r^2 + \ell(\ell+1)\xi_h^2 - 2\xi_r\xi_h - \xi_h^2]\rho r^4$$

$$F_{2,n,\ell} = \xi_h^2\rho r^4$$

where ξ_r and ξ_h are the radial and horizontal components of the displacement vector respectively and ρ is the density, while:

$$v_{2j+1,\ell} = v_{2j-1,\ell}\left(-\frac{2j+1}{j}\frac{\ell-j}{2\ell+2j+1}\right)$$

starting from $v_1 = 1$ for $j = 0$.
Equation (3) constitutes the basis for the 1.5-dimensional inversion. Now, the original inverse problem (equation 1) has been decomposed into a series of 1-dimensional independent inversions for each a-coefficient.
It is useful to notice that for high degree p-modes ($\ell \ll j$), $|F_{1,n,\ell}| \gg F_{2,n,\ell}$ so the angular velocity could be quickly found ignoring $F_{2,n,\ell}$ in equation (3). This approximation fails below $0.4R\odot$ leading to wrong solutions in the core.

2. INVERSION TECHNIQUE

In order to invert the data, we used the localized averaging kernel method (Backus & Gilbert 1970) which allows us to estimate a localized weighted average of angular velocity $\bar{\Omega}(r_0)$ at selected target radii r_0's by means of a linear combination of all the data:

$$\bar{\Omega}(r_0) = \sum_{i}^{M} c_i(r_0) a_i = \sum_{i=1}^{M} c_i(r_0) \int_0^R \mathcal{K}_i(r)\Omega(r)dr$$

where $c_i(r_0)$ are the inversion coefficients and

$$K(r_0, r) = \sum_{i=1}^{M} c_i(r_0)\mathcal{K}_i(r)$$

are the averaging kernels.
Because of the ill-conditioned nature of the inversion problem, it is necessary to introduce a regularization procedure to produce (for some choice of the trade-off parameter) a well localized averaging kernel to obtain $\bar{\Omega}(r_0)$ as close as possible to $\Omega(r = r_0)$.

We applied the method in the variant form, as described in Pijpers and Thompson (1992), known as SOLA (Subtractive Optimally Localized Averaging), making attempts to fit the averaging kernel to a Gaussian function of appropriate width, centred at the target radius. The trade-off parameter is rescaled at each target location to keep constant the width of the averaging kernels and to obtain more localized resolving kernels closer to the nominal concentration point. Following the prescription of equation (3) we performed 1-dimensional SOLA inversion of each set of a_{2j+1} coefficients (for $j = 0, 8$) to determine the expansion functions $\widetilde{\Omega}_{2j+1}(r)$, whose combination according to equation (2) leads to $\Omega(r, \mu)$:

$$\Omega(r, \mu) = \widetilde{\Omega}_1(r) + \widetilde{\Omega}_3(r)\frac{dP_3(\mu)}{d\mu} + \widetilde{\Omega}_5(r)\frac{dP_5(\mu)}{d\mu} + ... \quad (5)$$

3. RESULT OF INVERTING SOI-MDI DATA

We infer the rotational behaviour of the Sun by inverting a set of more than 30,000 of p-mode splitting coefficients. These were obtained from the first 144 days of data from the medium-ℓ programme of SOI-MDI in 1996 (Kosovichev et al. 1997). The degree ℓ ranges from 1 to 250, and the frequency from 954 μHz to 4556 μHz.

The inferred rotation rate is shown in Fig. 1 where the points indicate the angular velocity at various depths calculated at the equator, and at latitudes of 30°, 60° and 75°. The radial spatial resolution is assumed to be the half-width at half-maximum of the resolving kernels, while the vertical bars are related to the standard deviations of the observations. In Fig. 2 colours and contours indicate the isorotation surfaces.

We confirm the previous findings (Tomczyck et al. 1995) that the latitudinal differential rotation observed at the surface more or less persists throughout the convection zone, while the radiative interior rotates almost rigidly at a rate of about $430nHz$. At low latitudes the angular velocity, through the largest part of the convection zone, decreases with the radius while at high latitudes increase inwards. The near-surface behaviour agrees with the observed surface rotation rate.

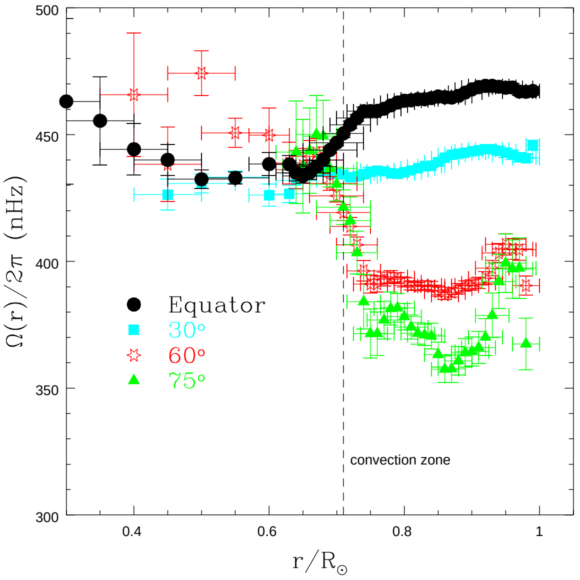

Figure 1. Differential rotation at four latitudes as obtained by a 1.5 dimensional SOLA inversion of the SOI-MDI data. The base of the convection zone is indicated by the dashed line.

By using the SOI-MDI data, we have been able to determine solar rotation closer to the poles than in the previous investigations. The data show that the Sun's angular velocity is significantly slower at high latitudes than that inferred from earlier helioseismic inversions.

The tachocline, the transition layer from the latitudinally dependent rotation to the nearly independent one (Spiegel & Zahn 1992), is shown very clearly if one performs a 1-dimensional inversion of the a_1, a_3 and a_5 coefficients for determining the expansion functions $\widetilde{\Omega}_1(r)$, $\widetilde{\Omega}_3(r)$, $\widetilde{\Omega}_5(r)$. The tachocline at the equator (Fig. 3 and Fig. 4) appears to be located mostly in the radiative zone in a thin layer $0.05R_\odot$ wide, centred at about $0.67R_\odot$. The tachocline layer at high latitudes seems to be wider, but certainly

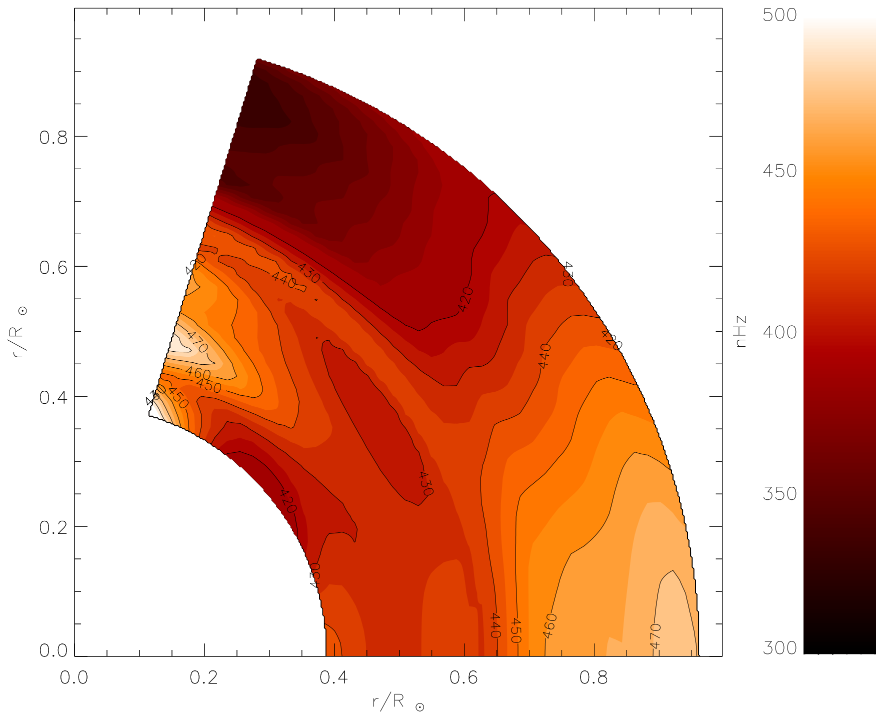

Figure 2. *Rotation rate in the Sun obtained by 1.5-dimensional SOLA inversion of SOI-MDI data.*

less than $0.1R_\odot$. The location and tickness of the tachocline, here determined, which are of great interest for solar dynamo theorists, are consistent with the determinations by Corbard et al. (1997).

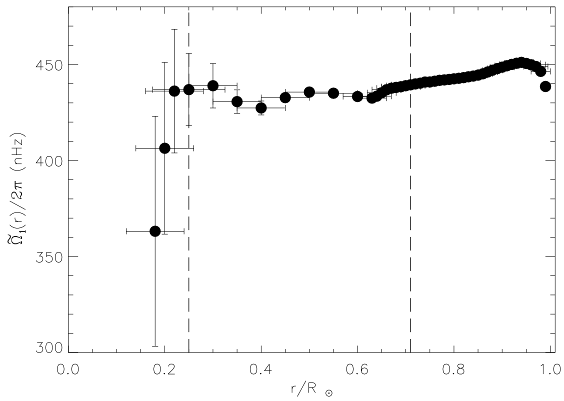

Figure 3. $\widetilde{\Omega}_1(r)$ *as deduced performing a 1-dimensional SOLA inversion of* a_1*-coefficient. It is directly related to the angular momentum. The vertical dashed lines indicate approximately the solar core and the base of the convection zone.*

The function $\widetilde{\Omega}_1(r)$, whose behaviour is shown in Fig. 3, is directly related to the angular momentum $\Im$:

$$\Im = \int_0^{M_\odot} r^2 dM_r \int_0^1 (1-\mu^2)\Omega(r,\mu)d\mu =$$

$$= \frac{2}{3}\int_0^{M_\odot} \widetilde{\Omega}_1 r^2 dM_r. \tag{6}$$

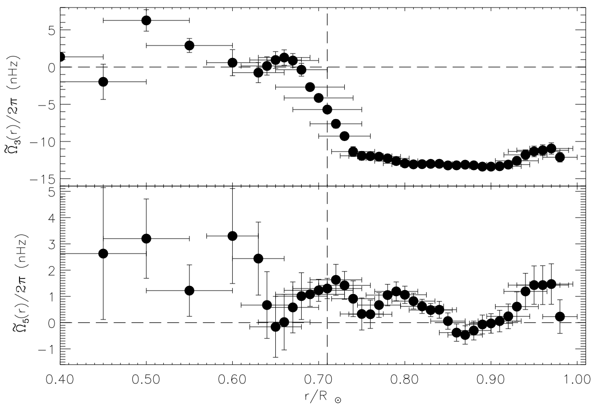

Figure 4. *1-dimensional SOLA inversion of* a_3 *and* a_5 *SOI-MDI coefficients. The vertical dashed line indicates the approximate base of the convection zone* $r = 0.71R_\odot$.

Moreover, unless magnetic fields play an important role in controlling the core's rotation, the angular velocity in the inner regions should not depend on latitude, and, according to equation (5), we expect $\Omega(r) \simeq \widetilde{\Omega}_1(r)$. The profile of $\widetilde{\Omega}_1(r)$ suggests a slowly rotating core, but unfortunately, the uncertainties in measurements preclude any firm conclusion.

4. ROTATION OF THE SUN'S CORE

To infer accurately the rotation in the deepest interior, it is necessary to invert a set of data with more accurate splittings of the lowest degree modes ($\ell = 1-4$). We use data sets, suitable for this purpouse, obtained by the groundbased networks Bi-

SON, IRIS and GONG (Chaplin et al. 1996; Lazreck et al. 1996; Gavryusev & Gavryuseva 1998). The first set inverted includes individual splittings with $1 \leq \ell \leq 4$ generated from Doppler velocity residuals collected for sixteen months by BiSON (Chaplin et al. 1996) between May 1993 and August 1994.
The second set comprises IRIS splittings, with $1 \leq \ell \leq 3$, computed as weighted averages over the radial orders n by using measurements by Lazreck et al. (1996) and Gizon et al. (1997) combined with some additional more recent data (Fossat, 1998).
The third set consists of rotational splittings ($1 \leq \ell \leq 4$) averaged over all n calculated, by Gavryuseva & Gavryusev (1998), on 830 days of observations of GONG instruments starting from 10 June 1995.
Finally, we compare these results with the solutions obtained using the very recent and accurate low-ℓ data ($\ell = 1, 2$), obtained by the GOLF instrument on SOHO after 538 days of uninterrupted measurements starting from April 1996.
Since the dominant contributions to the frequency splittings come from the outer parts of the Sun, in our analysis the splittings of the lowest degree modes have been combined with SOI-MDI a_1 splitting coefficients for the highest degree modes, for all the sets of low-ℓ data we used.

Unfortunately observations are still not in mutual agreement, so these inversions give conflicting results in the core, as is shown in Fig. 5, where the points indicate the angular velocity at various depths. The radial spatial resolution, for clarity not drawn in the figure, is fixed at $\Delta r = 0.1 R_\odot$. The indipendent sets of observations obtained by IRIS, GONG, GOLF lead to the conclusion that the Sun's core is in a state of rotation slightly faster than that observed at the surface in contradiction with the BiSON's data inversion which indicates a central angular velocity even slower than the surface polar angular velocity. The disagreement might derive from the difference in data analysis procedures.

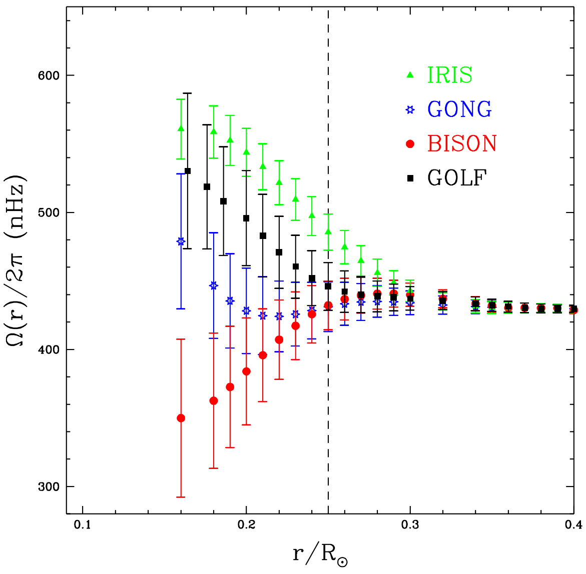

Figure 5. Rotation of the sun's core as deduce by inversion of the BiSON (filled circles), IRIS (filled triangles), GONG (starred symbols) and GOLF (filled squares) sets of lowest degree splittings ($\ell = 1-4$), all combined with SOI-MDI higher degree data set.

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